RESET CONTROL SYSTEMS: STABILITY, PERFORMANCE AND APPLICATION

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Dedicated to my parents and my wife

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ABSTRACT

RESET CONTROL SYSTEMS: STABILITY, PERFORMANCE AND APPLICATION

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Linear time invariant (LTI) control design is the most widely applied control design technique. But it has inherent limitation. In LTI feedback control systems, high-frequency loop gain is linked to both low-frequency loop gain and stability margins through Bode's gain-phase relationship. This linkage results in trade-offs among competing performance specifications which LTI control design can not provide remedy. Reset control design is a great candidate of improving this limitation in LTI control design. The basic idea of reset control is to reset the state of a linear controller to zero whenever its input meets a threshold. Such a reset controller is introduced in feedback control systems with the aim of providing better trade-offs between competing specifications than could be achieved using linear controllers.

There have been some successful experimental applications of reset control technique. These examples demonstrate that reset control has the potential benefit of improving the inherent performance trade-offs in LTI control, while inherits the advantages of LTI control design. However, during the past three decades, there is a lack of theoretical results for reset control systems. For example, none of the experimental reset control applications can provide formal proof of stability and theoretic analysis of performance. Dedicated to solving this problem, this dissertation performs a complete theoretic analysis of the stability and time-domain performance of reset control systems. A complete set of theoretic results on the stability and time-domain performance are developed. These results build a solid theoretic base for the further and wider application of reset control system. Also, reset control design technique is applied successfully for the speed control of a rotational mechanical system. This experiment demonstrates the benefit of reset control design over LTI control design and the effectiveness of the theoretic results developed in this dissertation.

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CHAPTER 1

INTRODUCTION

1.1 Background

Linear time-invariant (LTI) feedback control design is the most mature and widely applied control design technique. But LTI control design has its inherent limitation. In LTI feedback control systems, high-frequency loop gain is related to both low-frequency specifications and stability margins through Bode's gain-phase relationship. This relationship (referred to as the "cost of feedback" [17]) results in trade-offs among competing performance specifications which are unavoidable in LTI control. To further illustrate this problem, consider the typical LTI control system as shown in Figure 1.1, where $C(s)$ and $P(s)$ are the transfer functions of controller and plant respectively and signals $r(t)$, $y(t)$, $n(t)$ and $d(t)$ represent reference input, output, sensor noise and disturbance respectively. The following are typical genetic specifications for control design.

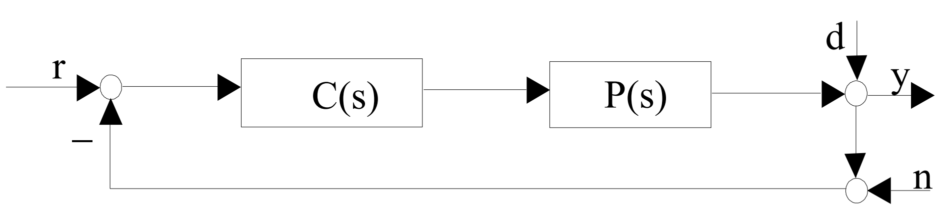

Figure 1.1 Typical LTI feedback control system

1. In the low-frequency band, we require the sensitive function,

$$S(j\omega) = \frac{1}{1 + |P(j\omega)C(j\omega)|},$$

to be small, $|S(j\omega)| < 1$, for disturbance rejection. This requires that the open-loop magnitude $|P(j\omega)C(j\omega)|$ be high in low-frequency band.

2. In high-frequency band, we require the complementary sensitive function,

$$T(j\omega) = \frac{|P(j\omega)C(j\omega)|}{1 + |P(j\omega)C(j\omega)|},$$

to be small, $|T(j\omega)| < 1$, for sensor noise suppression. This requires that $|P(j\omega)C(j\omega)|$ be small in high-frequency band.

3. Enough stability margin.

The following example clearly explains how the trade-offs among these performance specifications happen. Figure 1.2 shows the magnitude and phase of the open-loop transfer functions $P(j\omega)C(j\omega)$ of two LTI control designs. To further suppress the high-frequency sensor noise of linear design 1, we decrease its high-frequency loop gain and get linear design 2. According to Bode's gain-phase relationship, linear design 2 will have larger phase lag in the cross-over frequency range. So the cost of linear design 2 is that it has a smaller phase margin than linear design 1 as indicated in Figure 1.2. This trade-off is unavoidable in LTI control.

This limitation has motivated researchers to consider various forms of nonlinear control. A straightforward approach of improving the inherent limitation in LTI design is to adopt nonlinear non-smooth filters which are able to circumvents the frequency domain restriction of LTI filters, governed by Bode's gain-phase relationship

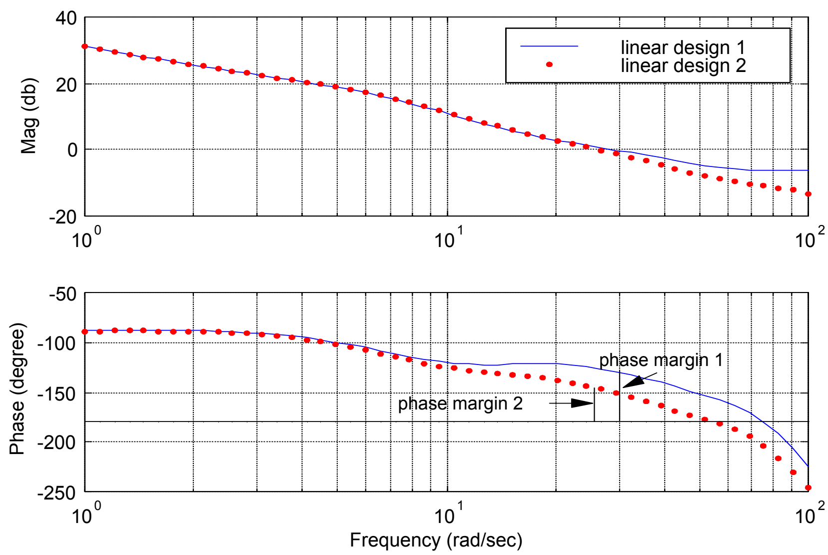

Figure 1.2 Illustration of "cost of feedback"

[44], [45]. The design and analysis of such kind of these nonlinear filters are normally based on the describing function approximation [14], [16]. These nonlinear filters are targeted to have a describing function approximations with much smaller phase lag and same magnitude slope characteristics compared to the corresponding LTI filters. From the above illustration of the "cost of feedback" problem, we can see that it is possible to improve the performance trade-offs by using these nonlinear filters in place of the corresponding linear filters. Such nonlinear filters started with Lewis servo [46] and the Kalman nonlinear gain element [47]. Other nonlinear non-smooth filters include driven limiter [48] and the split-path-nonlinear filter [49]. Another interesting example is the resetting virtual absorbers for vibration control by Bupp and Bernstein [22]. Our interest is on the nonlinear integrator which is introduced by Clegg in 1958 [1]. It is basically a linear integrator whose state resets to zero

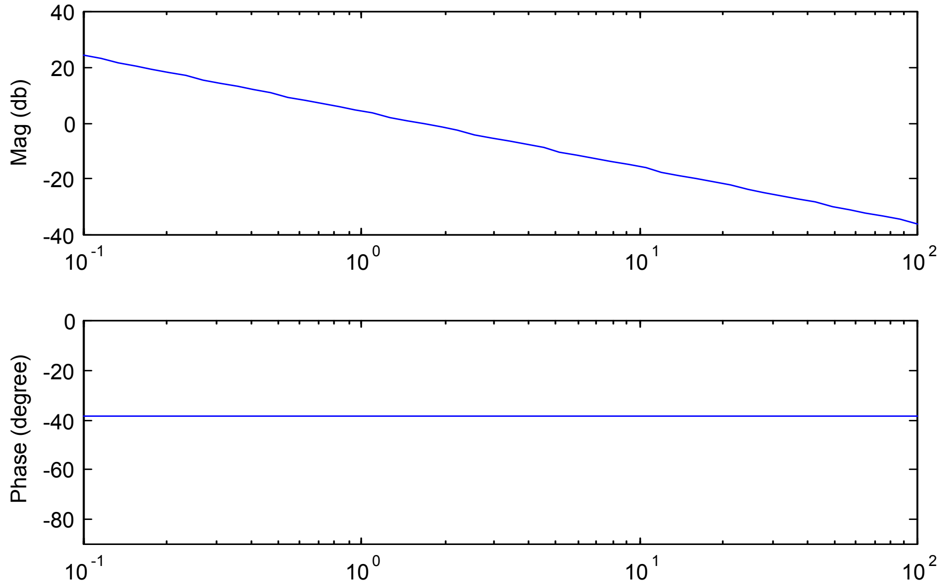

Figure 1.3 Describing function of Clegg-integrator

when its input crosses zero. This so-called Clegg-integrator can be described with the following impulsive differential equations [18]:

$$\begin{aligned} \dot{x}_c(t) &= u(t); \qquad u(t) \neq 0 \\ x_c(t^+) &= 0; \qquad u(t) = 0, \end{aligned} \tag{1.1}$$

where $u(t)$ and $x_c(t)$ denote the input and state of Clegg-integrator respectively. Clegg-integrator was shown to have a describing function with similar magnitude to the frequency response of a linear integrator but with $51.9°$ less phase lag (see Figure 1.3). This feature makes it a great candidate for improving the performance trade-offs in LTI control design.

The Clegg integrator was later generalized by Krishnan and Horowitz to a so-called first-order reset element (FORE) [2]. FORE simply extended the Clegg concept

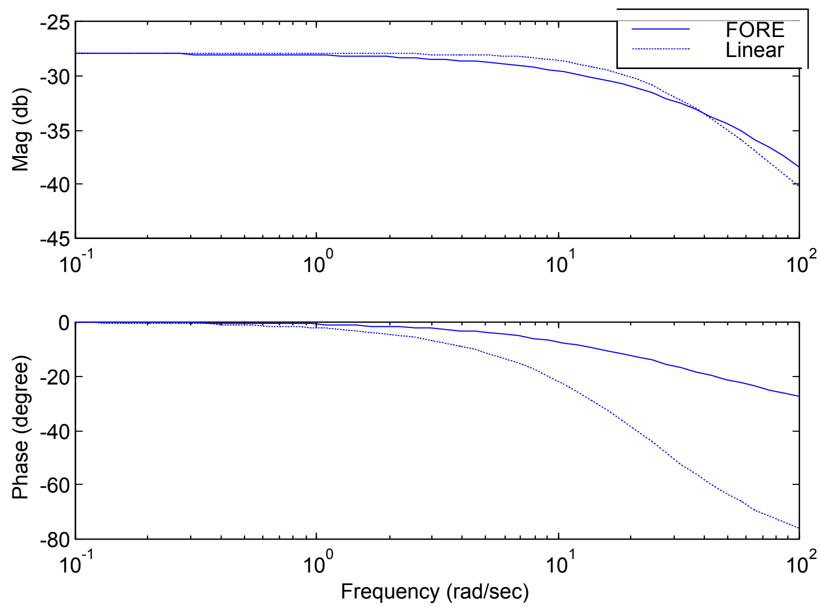

Figure 1.4 Describing function of FORE (b=25) and Bode-plot of 1/(s+25)

to a lag filter $1/(s+b)$, where b is called FORE's pole. It can be described with the impulsive differential equations:

$$\begin{aligned} \dot{x}_c(t) &= -bx_c(t) + u(t); \quad & u(t) \neq 0 \\ x_c(t^+) &= 0; & u(t) = 0, \end{aligned} \tag{1.2}$$

where $u(t)$ and $x_c(t)$ denote the input and state of FORE respectively. The describing function of FORE when $b = 25$ is shown in Figure 1.4 compared to the frequency response of the linear filter $1/(s + 25)$. It shows that they have similar magnitude while FORE has much less phase lag than the linear filter in high-frequency. The implication is that we may improve performance trade-offs by using FORE in place of corresponding linear filter.

1.2 Description of Reset Control System

This dissertation is focused on the reset control system which was introduced as a possible means to mitigate this limitation. The basic idea of reset control is to reset the state of a linear controller to zero whenever its input meets a threshold. Typical reset controllers include the Clegg-integrator [1] and first-order reset element (FORE) [3] introduced in preceding section. The structure of the reset control system under consideration is shown in Figure 1.5, where signals $r(t)$, $y(t)$, $e(t)$, $n(t)$ and $d(t)$ represent reference input, output, error signal, sensor noise and disturbance respectively and $L(s)$ is the transfer function of the linear loop which includes both the linear controller $C(s)$ and the plant $P(s)$, i.e. $L(s) = C(s)P(s)$. In the absence of resetting, the FORE behaves as the linear filter $1/(s + b)$. In this case, we refer to the resulting linear, closed-loop system as the *base linear system*.

The FORE element can be described by the first-order impulsive differential equation [18]:

$$\begin{aligned} \dot{x}_c(t) &= -bx_c(t) + e(t); \;\; e(t) \neq 0 \\ x_c(t^+) &= 0; \qquad\qquad\qquad\quad e(t) = 0 \end{aligned} \tag{1.3}$$

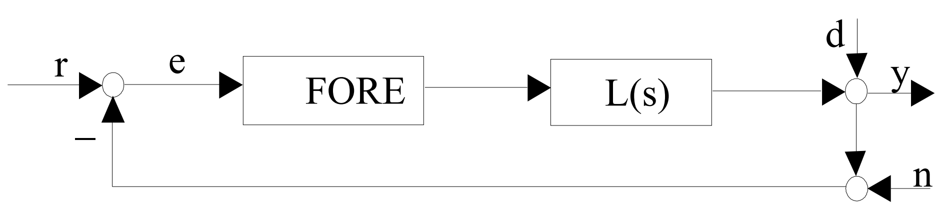

Figure 1.5 Block diagram of reset control system

where $x_c(t)$ is the state of FORE and $e(t) = r(t) - n(t) - d(t) - y(t)$ is the error. The time instants when $e(t) = 0$ are called *reset times* and the set of reset times I is defined as

$$I = \{t_i \mid e(t_i) = 0, t_i > t_{i-1}, i = 1, 2, ...\},$$

where t_i is called the i th reset time. The transfer function $L(s)$ denotes the linear loop which can be described by the state equations

$$\begin{aligned} \dot{x}_p(t) &= Ax_p(t) + Bx_c(t) \\ y(t) &= Cx_p(t) + d(t) \end{aligned} \tag{1.4}$$

where $x_p(t) \in \Re^n$ is the plant states and $\{A, B, C\}$ is a minimal realization of $L(s)$. From (1.3) and (1.4) the reset control system in Figure 1.5 can then be described by the following impulse differential equation [18]

$$\begin{aligned} \dot{x}_p(t) &= Ax_p(t) + Bx_c(t) \\ \dot{x}_c(t) &= -Cx_p(t) - bx_c(t) + w(t); \quad t \notin I \\ x_c(t^+) &= 0; \qquad\qquad\qquad\qquad\quad t \in I \end{aligned} \tag{1.5}$$

where $w(t) = r(t) - n(t) - d(t)$.

In the case that there is no reset actions, reset control system (2.1) reduces to the following linear system which is referred as its *base linear system.*

$$\begin{aligned} \begin{bmatrix} \dot{x}_{pl}\ (t) \\ \dot{x}_{cl}\ (t) \end{bmatrix} &= A_{cl} \begin{bmatrix} x_{pl}(t) \\ x_{cl}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ w(t) \end{bmatrix}, \\ \begin{bmatrix} x_{pl}(0) \\ x_{cl}(0) \end{bmatrix} &= \begin{bmatrix} x_p(0) \\ x_c(0) \end{bmatrix}, \end{aligned} \tag{1.6}$$

where

$$A_{cl} = \begin{bmatrix} A & -B \\ -C & -b \end{bmatrix}.$$

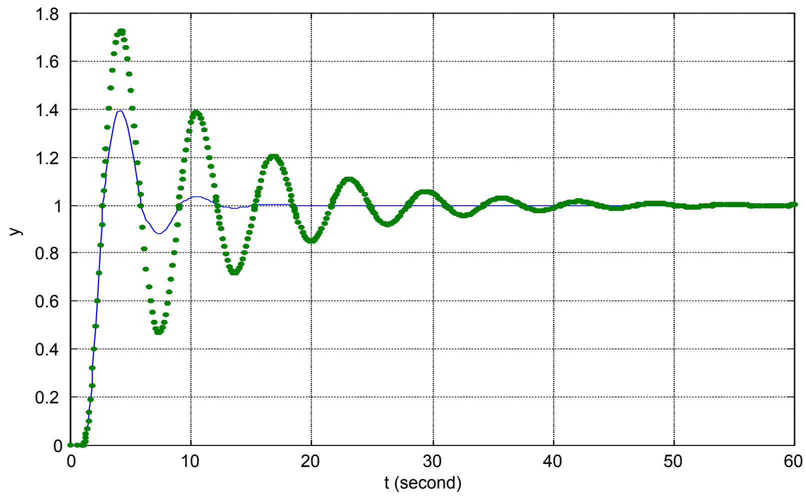

Figure 1.6 Comparison of step responses for the reset control system (solid) and its base linear system (dot)

1.3 Motivation

Despite of its simplicity, the above reset control system was shown to have the potential benefit of providing improved trade-offs in feedback control. The describing functions of Clegg integrator and FORE give a reasonable explanation of the source of potential benefit. The potential benefit of reset control has been exemplified by the earlier work in [1]-[4]. Recently, the benefit of reset control was also confirmed experimentally in [5] and [6] where a FORE was used in the design of a tape-speed control system. In Chapter 5 of this dissertation, the experiment of control system design for a rotational mechanical system also verifies the benefit of reset control design. As an illustration, we now repeat a simple example originally presented in [10]. In this example, $L(s)$ is

$$L(s) = \frac{s+1}{s(s+0.2)}$$

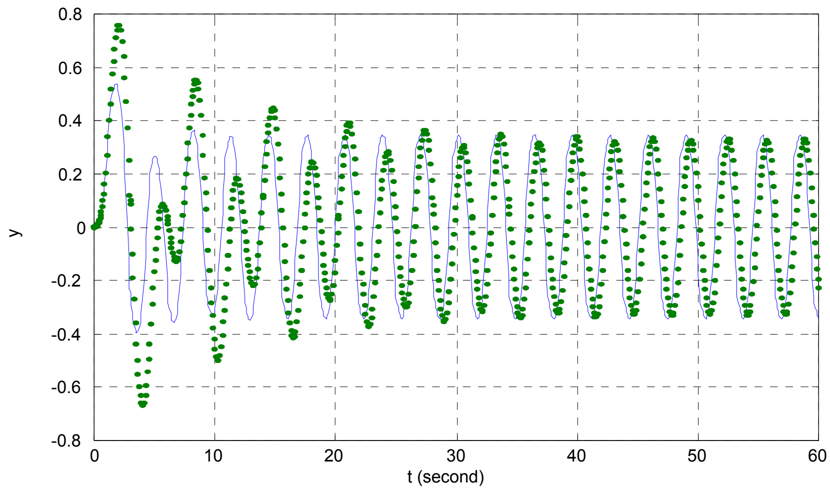

Figure 1.7 Comparison of responses to 2 rad/sec sinusoidal sensor noise for the reset control system (solid) and its base linear system (dot)

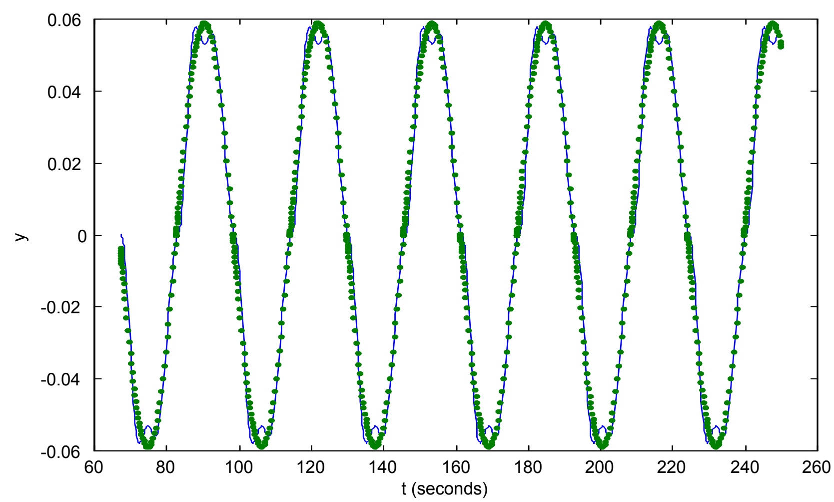

Figure 1.8 Comparison of response to 0.2 rad/sec sinusoid disturbance for the reset control system (solid) and its base-linear system (dot)

and FORE's pole is $b = 1$. Figure 1.6 shows that this reset control system achieves a 40% reduction in step response overshoot compared to its base linear system. Figure 1.7 and 1.8 shows that their responses to 2 rad/sec sinusoidal sensor noise and 0.2 rad/sec sinusoidal disturbance are similar. The implication is that compared to its base linear system, a reset control system appears to have much less overshoot in its step response while maintaining similar high-frequency sensor-noise suppression and low-frequency disturbance rejection. Since overshoot is closely related to stability margin, a smaller overshoot generally implies a larger stability margin. In LTI design, a larger stability margin is normally achieved in the cost of sacrificing its performance of sensor-noise suppression or disturbance rejection. This is not seen in reset control which implies its potential of improving the performance trade-offs in LTI design.

Despite of this potential benefit, there is a lack of theoretic results in stability and performance analysis of reset control system. The analysis in [1] was limited to the describing function of Clegg integrator, and [2] and [3] provided some design guidelines without formal proof. [8] and [9] developed results for asymptotic stability limited to some special reset control systems with second-order plants. Although [4] and [5] developed some general stability results, they are too conservative to be applicable. For example, although the reset control design in [6] shows very good performance, there is no formal proof of stability and steady-state behavior. Dedicated to solving this problem, this dissertation builds the theoretical foundations of stability and time-domain performance for reset control. It is believed that these results will further enable the application of reset control in engineering applications.

In this dissertation, we are faced with the following major challenges in the research of reset control. To date there are no results available.

1. Theoretic analysis of stability under arbitrary bounded input.

2. Theoretic analysis of steady-state behaviors such as zero steady-state error.

3. Theoretic analysis of performance.

We theoretically establish basic properties of reset control systems including stability, transient and steady-state performance and apply these results to some experimental setups of reset control design. Some of the results have appeared in [10]-[12].

Finally, we remark that reset control resembles a number of popular nonlinear control strategies including relay control ([36], [37]), variable structure and sliding mode control ([38], [39], [42], [43]) and switching control ([28], [40], [41]). A common feature to all of these is the use of a switching surface to trigger change in the control signal. The difference in reset control is that the same control law is used on both sides of the switching surface. A change takes place on a fixed surface wherein the controller states are reset to zero. So, compared to these popular nonlinear control strategies, reset control may be easier to design and simpler to implement. There is no need to design switching surface and different control laws. As we will demonstrate later, reset control design is as easy as linear control design. Since reset action can be modeled as the injection of judiciously-timed, state-dependent impulses into an otherwise linear feedback system, reset control system is alike with system with impulse effect ([18], [19], [29]) or system with jump [34]. This analogy is evident when reset control systems were modeled by impulsive differential equations; e.g., see [18]. However, for our specific case of reset control system, the existing results for impulse differential equations are too conservative to be applicable. For example, with these results we are not able to provide stability guarantee for the experimental reset control system in [6].

1.4 Dissertation Contributions

This dissertation has made the following contributions:

1. Developed a sufficient condition which guaranteed BIBO stability of reset control system even in the face of implementation errors. Furthermore, this condition was demonstrated to be non-conservative.

2. Developed a general result regarding the steady-state behavior of reset control system. Additional results such as asymptotic stability and zero steady-state errors were derived as the specific cases of this result.

3. Developed a so-called "input decomposition" principle for reset control systems which simplified the analysis of system response.

4. Characterized the maximum overshoot, rise time and settling time for step response. For a specific class of reset control systems of reset control systems with second-order plants, we explicitly computed the values of maximum overshoot, rise time and settling time.

5. For the reset control systems with second-order plants, we developed a sufficient and necessary condition for asymptotic stability and a sufficient condition for BIBO stability.

6. We applied reset control design technique for the speed control of a rotational mechanical system and performed theoretic analysis of this reset control system.

1.5 Dissertation Outline

The structure of this dissertation is as follows: Chapter 2 discusses stability of reset control systems. A sufficient condition is developed which guarantees the BIBO stability of reset control system even in the face of implementation errors. As a by-product of this result, a general result regarding the steady-state behavior of reset control system is derived. Additional results such as asymptotic stability and zero steady-state errors are established as the specific cases of this result. Chapter 3 focuses on the performance issue. We give a result which guarantees zero steady-state error for reset control systems. Also, the so-called "input decomposition" principle for reset control systems is developed. In Chapter 4 we study the reset control systems with second-order plants. A sufficient and necessary condition for asymptotic stability and a sufficient condition for BIBO stability are developed. The step response is analyzed and the values of maximum overshoot, rise time and settling time are explicitly computed for a specific class of reset control systems. In Chapter 5, reset control design technique is applied for the speed control of a rotational flexible mechanical system. We also perform theoretic analysis of stability and steady-state behaviors for this reset control system. Finally, in Chapter 6, conclusions and future work are presented.