Stability of A Reset Control System Under Constant Inputs [1]

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Abstract

Reset controllers are standard linear compensators equipped with mechanism to instantaneously reset their states. With respect to pure linear control, there is evidence that this reset action is capable of improving control system tradeoffs. This paper's objective is to analyze the stability of a particular example of reset control system when excited by constant inputs. Our main result shows that the equilibrium point of the closed-loop dynamics is asymptotically stable.

1 Introduction

The particular type of control system considered in this paper is shown in Figure 1 and utilizes the so-called first-order reset element (FORE) introduced in [1]. This FORE element can be described by

$$\begin{aligned} \dot{x}_c &= -bx_c + e; \quad e \neq 0 \\ x_c &= 0; \quad e = 0 \\ u &= x_c; \end{aligned} \tag{1}$$

where x_c is the controller state, u is the controller output and $b > 0$ is the FORE's pole. In the absence of resetting, the FORE behaves as the linear system $\frac{1}{s+b}$. The plant in Figure 1 is purposely chosen so that the associated linear complementary sensitivity function has standard (stable) second order form

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where $\zeta > 0$ and $\omega_n > 0$ denote the damping ratio and natural frequency respectively. It has been shown in [1], and more recently in hardware experiments [2], that reset control systems of this type exhibit improved tradeoffs (in comparison to linear control) between competing objectives such as disturbance rejection, transient performance and high-frequency sensor noise attenuation. However, [1] and [2] lacked any stability analysis and this paper, in the same vein as some of our previous research [3], aims to provide some. Specifically, we will show in the next section that the equilibrium point for the reset control system in Figure 1 is asymptotically stable if r, d or n is a constant signal. For example, for constant reference signal r and any initial condition, the tracking error e asymptotically converges to zero.

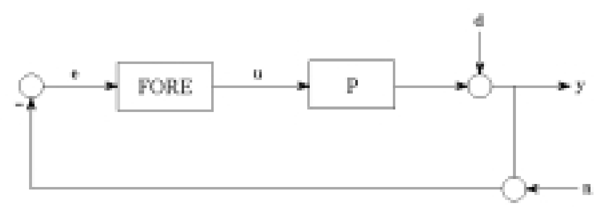

Figure 1: Block Diagram of a Reset System

2 Stability of the Reset Control System

Let the open-loop plant in Figure 1 have state-space realization

$$\begin{aligned} \dot{x}_p &= Ax_p + Bu \\ y &= Cx_p + d \end{aligned} \tag{2}$$

where x_p is the plant state;

$$A = \begin{bmatrix} -2\zeta\omega_n & 1 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ b \end{bmatrix}; \quad C = \begin{bmatrix} \omega_n^2 & 0 \end{bmatrix}.$$

For simplicity we focus on the response to constant disturbances and take $r = 0$, $n = 0$ and $d(t) \equiv d_0$. Using (1) and (2), we can then describe this control system by the *resetting differential equation* (see [4])

$$\begin{aligned} \dot{x} &= A_{c\ell}x - \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}' d_0; \quad x \notin M_{d_0} \\ \Delta x &= (A_R - I)x; \quad x \in M_{d_0} \end{aligned} \tag{3}$$

where $x = [x_p' \;\; x_c]'$ is the closed-loop state,

$$A_{c\ell} = \begin{bmatrix} A & B \\ -C & -b \end{bmatrix}; \quad A_R \doteq \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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and M_{d_0} is the *set of reset states*

$$M_{d_0} = \{\xi : C_y \xi = -d_0\}$$

where

$$C_y = \left[\begin{array}{ccc} \omega_n^2 & 0 & 0 \end{array} \right].$$

Assumption 1: Given an initial condition x_0 and associated unique solution $\phi(x_0, t)$ to (3), the set of reset times $\{t > 0 : C_y \phi(x_0, t)\}$ is an unbounded discrete subset of $\mathbb{R}^+$.

The equilibrium state x_e for (3) is

$$x_e = A_{c\ell}^{-1} \left[\begin{array}{ccc} 0 & 0 & 1 \end{array} \right]' d_0 = \left[\begin{array}{ccc} \frac{-d_0}{\omega_n^2} & \frac{-2\zeta d_0}{\omega_n} & 0 \end{array} \right]'.$$

To establish its asymptotic stability, we consider the quadratic Lyapunov candidate $V(x) = x'Px$ and state a special case of the general stability result in Theorems 13.1 and 13.2 of [4].

Theorem 1: (see [5] for proof) *Under Assumption 1, the equilibrium state x_e is asymptotically stable if there exists a positive-definite symmetric matrix P such that*

$$x' \left(A_{c\ell}' P + P A_{c\ell}\right) x < 0; \quad x \notin M \tag{4}$$

and

$$x' \left(A_R' P A_R - P\right) x \leq 0; \quad x \in M \tag{5}$$

where

$$M \doteq \{\xi : C_y \xi = 0\}.$$

Our main result claims that the conditions in Theorem 1 are always satisfied.

Theorem 2: *Under Assumption 1, the equilibrium state x_e is asymptotically stable for all positive b, ζ and ω_n.*

Proof: To begin the proof, let $\Theta = \left[\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]'$. Since the columns of Θ span the nullspace of C_y, then

$$M = \{\Theta \xi : \xi \in \mathbb{R}^2\}. \tag{6}$$

Using (6), we can write (5) as

$$\Theta' \left(A_R' P A_R - P\right) \Theta = \left[\begin{array}{cc} 0 & -p_{23} \\ -p_{23} & -p_{33} \end{array} \right] \leq 0.$$

Since P is positive-definite, (5) holds if $p_{23} = 0$. Without loss of generality, assume $p_{33} = 1$. Then, (5) holds if there exists a $\beta \in \mathbb{R}$ such that

$$\left[\begin{array}{ccc} 0 & 0 & 1 \end{array} \right] P = \left[\begin{array}{ccc} \beta & 0 & 1 \end{array} \right]. \tag{7}$$

From the Kalman-Yakubovich Lemma; e.g., see [6], (4) and (7) hold for some positive-definite symmetric matrix P if there exists a $\beta \in \mathbb{R}$ such that the transfer function

$$h_\beta(s) \doteq \left[\begin{array}{ccc} \beta & 0 & 1 \end{array} \right] (sI - A_{c\ell})^{-1} \left[\begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right]$$

is strictly positive-real (SPR). The remainder of the proof deals with finding such a β and these details can be found in [5]. □

3 Conclusions

In this paper we have shown that the reset control system in Figure 1 is asymptotically stable when either r, d or n is a constant. Since the associated linear system also enjoys this same property, then the real benefit of our work comes in combining it with the studies in [1] and [2] to conclusively conclude that such reset control system designs are indeed well-behaved to constant inputs. Such analysis was previously missing. Finally, the concept behind Theorems 1 and 2 can be easily generalized to higher-order plants. However, the strong results of Theorem 2 will not carry over.

References

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