Stability and Asymptotic Performance Analysis of a Class of Reset Control Systems

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Abstract

Bode's gain-phase relationship places a hard limitation on performance tradeoffs in linear, time-invariant feedback control systems. It has long been suggested that reset control has the potential to improve this situation. Recent experimental studies support this claim. This paper focuses on the analysis of such reset control systems which has been missing in this past work. Specifically, we give results on bounded-input bounded-output stability, asymptotic stability and steady-state performance. These results are applied to an experimental demonstration of reset control of a flexible mechanism.

1 Introduction

It is well-appreciated that Bode's gain-phase relationship [1] places a hard limitation on performance tradeoffs in linear, time-invariant (LTI), feedback control systems. Specifically, the need to minimize the open-loop high-frequency gain often competes with required high levels of low-frequency loop gains and stability margin bounds. Our focus on reset control systems is motivated by its potential to improve this situation as demonstrated theoretically in [2][4] and by simulations and experiments [3]-[6].

The basic concept in reset control is to reset the state of a linear controller to zero whenever its input meets a threshold. Typical reset controllers include the so-called Clegg integrator [7] and first-order reset element (FORE) [3]. The former is a linear integrator whose output resets to zero when its input crosses zero. The latter generalizes the Clegg concept to a first-order lag filter. In [7], the Clegg integrator was shown to have a describing function similar to the frequency response of a linear integrator but with only 38.1° phase lag. A FORE was shown to have similar feature while providing a further design freedom when compared with Clegg-integrator ([4] and [6]). In our study, we adopt the FORE reset mechanism in feedback interconnection with a linear system to obtain the so-called *reset control system* shown in Figure 1. The signals r, y, e, n and d in Figure 1 represent reference input, output, error signal, sensor noise and disturbance, respectively, and $L(s)$ denotes the linear loop consisting of the plant and any linear compensation[5].

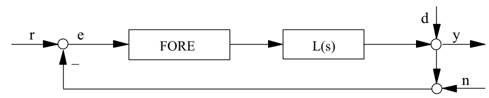

Figure 1: Block diagram of the servo system.

The objective of this paper is to provide a level of analysis missing in past work on reset control. The analysis in [7] was limited to describing functions while [3] and [4] ignored stability issues altogether. An application of small gain in [5] appears too conservative and could not validate the observed experimental performance in [6]. Motivated by this lack of results, this paper continues our recent work reported in a sequence of conference papers [8], [9], and [10]. In this paper, we introduce a condition, called the β *positive-real condition*, which, when satisfied, allows one to assert BIBO and asymptotic stability of the reset control system. Under this condition, we will also show that the reset control system inherits the steady-state tracking properties of an underlying linear control system. Very importantly, we will show that the β positive-real condition is satisfied for the experiment considered in [6], thus confirming the observed stability as well as demonstrating the applicability of our results.

Reset control action resembles a number of popular nonlinear control strategies including relay control [11], sliding mode control [12] and switching control [13]. A common feature to these is the use of a switching surface to trigger change in control signal. Distinctively, reset control employs the same (linear) control law on both sides of the switching surface. Resetting occurs when the system trajectory impacts this surface. This reset action can be alternatively viewed

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[4]This work provides an example of control specifications that can be achieved by reset control and not by linear feedback.

[5]The design of the reset control system in Figure 1 involves the selection of *both* the FORE's pole and some linear compensation in $L(s)$. This will be discussed in Section 5.

as the injection of judiciously-timed, state-dependent impulses into an otherwise LTI feedback system. This analogy is evident in the paper where we use impulsive differential equations; e.g., see [14] and [15], to model dynamics. Despite this relationship, we found existing theory on impulse differential equations to be either too general or broad to be of immediate and direct use. Finally, this connection to impulsive control helps to draw comparison to a body of control work [16] where impulses were introduced in an open-loop fashion to quash oscillations in vibratory systems.

The paper is organized as follows. In Section 2 we set-up a model to describe the reset control system in Figure 1 and identify a key underlying linear control system which we refer to as the *base-linear system*. Section 3 is central. It introduces this notion of β positive-realness and links it to BIBO stability. In Section 4 we again use the β positive-real condition to show that the base-linear system passes-on its steady-state performance properties to the reset control system. In Section 5, we apply these results to an experimental tape-speed control system described in [6].

2 Set-Up

In this paper we focus on the reset control system in Figure 1 where the first-order reset element (FORE) is described by the impulsive differential equation [14]:

$$\begin{aligned} \dot{x}_f(t) &= -bx_f(t) + e(t); \quad e(t) \neq 0 \\ x_f(t^+) &= 0; \qquad\qquad\qquad e(t) = 0 \end{aligned}$$

where x_f is its state, e is the system error and b the FORE's pole; see [3]. To avoid degeneration to a LTI system, we assume that the FORE continually resets. We collect these reset times in the unbounded set

$$I = \{t_i \mid e(t_i) = 0,\ t_i > t_{i-1} + \sigma,\ \sigma > 0,\ i = 1, 2, ..., \infty\}$$

where we assume that adjacent reset times are no closer than σ. This assumption is technically motivated by a desire to have closed-loop solutions continuable over $[0, \infty)$, but is also met when FORE is digitally implemented and the sampling period is a lower bound to σ.

A state-space description of the reset control system is:

$$\begin{aligned} \dot{x}_p(t) &= Ax_p(t) + Bx_f(t) \\ \dot{x}_f(t) &= -Cx_p(t) - bx_f(t) + w(t); \quad t \notin I \\ x_f(t^+) &= 0; \qquad\qquad\qquad\qquad\qquad t \in I \\ y(t) &= Cx_p(t) + d(t) \end{aligned} \tag{1}$$

where $\{A, B, C\}$ denotes a minimal realization of $L(s)$, $x_p(t) \in \Re^n$ and $w(t) \triangleq r(t) - n(t) - d(t)$ is the aggregate input signal. Given $(x_p(0), x_f(0))$, the solution to (1) is piecewise left-continuous on the intervals $(t_i, t_{i+1}]$. In the absence of resetting, (1) reduces to the following linear system:

$$\begin{bmatrix} \dot{x}_{pl}(t) \\ \dot{x}_{fl}(t) \end{bmatrix} \triangleq A_{cl} \begin{bmatrix} x_{pl}(t) \\ x_{fl}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ w(t) \end{bmatrix}; \tag{2}$$

where $x_{pl}(0) = x_p(0)$, $x_{fl}(0) = x_f(0)$ and where

$$A_{cl} \triangleq \begin{bmatrix} A & -B \\ -C & -b \end{bmatrix}.$$

We refer to this as the *base-linear system* and, in the sequel, we will show that it can pass on some of its properties, such as stability and asymptotic performance, to its associated reset control system.

3 BIBO Stability Analysis

In this section we analyze the BIBO stability of (1) which requires every bounded input[6] w to produce a bounded output y. To begin this analysis we apply the transformation

$$\begin{aligned} z_p(t) &\doteq x_p(t) - x_{pl}(t) \\ z_f(t) &\doteq x_f(t) - x_{fl}(t) \end{aligned} \tag{3}$$

to (1) to obtain:

$$\begin{aligned} \dot{z}_p(t) &= Az_p(t) + Bz_f(t) \\ \dot{z}_f(t) &= -Cz_p(t) - bz_f(t); \quad t \notin I \\ z_f(t_i^+) &= -x_{fl}(t_i); \qquad\qquad t \in I. \end{aligned} \tag{4}$$

As an intermediate step, we show that boundedness of z_p implies that y is bounded.

Lemma 1: *Assume A_{cl} is asymptotically stable and r, d and n are bounded. If z_p is bounded, then the output y is bounded.*

Proof: We have

$$\begin{aligned} |y(t)| &= |Cx_p(t) + d(t)| \\ &\leq |Cz_p(t)| + |Cx_{pl}(t)| + |d(t)|. \end{aligned}$$

Since A_{cl} is stable and w is bounded, then x_{pl} is bounded. Output y is thus bounded. □

Before we present our main result on BIBO stability, we need the following lemmas.

[6] A signal z is said to *bounded* if there exists a constant M such that $|z(t)| < M$ for all t.

Lemma 2: *If A_{cl} is asymptotically stable and w is bounded, there exists constants M_1 and M_2 such that $|z_f(t_i^+)| < M_1$ and $|Cz_p(t_i)| < M_2$ for $i = 1, 2, \ldots, \infty$.*

Proof: Because A_{cl} is asymptotically stable and w, then x_{fl} and x_{pl} are bounded. From (4), $z_f(t_i^+) = -x_{fl}(t_i)$. Therefore, there exists an M_1 such that $|z_f(t_i^+)| < M_1$ for $i = 1, 2, \ldots, \infty$. By definition, $Cz_p(t_i) = w(t_i) - Cx_{pl}(t_i)$. Since w and x_{pl} are bounded, then there exists an M_2 such that $|Cz_p(t_i)| < M_2$ for $i = 1, 2, \ldots, \infty$. □

The next is the well-known Meyer-Kalman-Yakubovich Lemma [17].

Lemma 3: *Let $Z(s) = h(sI - F)^{-1}g$ be a scalar transfer function where H is asymptotically stable. If $Z(s)$ is strictly positive-real[7], then there exist a symmetric positive-definite matrix P, a vector q, and a positive constant ε such that*

$$\begin{aligned} F^T P + PF &= -q^T q - \varepsilon P; \\ Pg &= h^T. \end{aligned}$$

Our next definition introduces a positive-real condition that is key in establishing the results of this paper.

Definition 1: The reset control system (1) is said to satisfy the β *positive-real condition* if there exists a $\beta \in \Re$ such that

$$h(s) \triangleq [\beta C \quad 1](sI - A_{cl})^{-1}[0 \ \cdots \ 0 \ 1]^T \qquad (5)$$

is strictly positive-real.

We now state a main result:

Theorem 1: *The reset control system* (1) *is BIBO stable if the β positive-real condition* (5) *is satisfied.*

Proof: Since $h(s)$ in (5) is strictly positive-real, then, from Lemma 3, there exists a positive-definite matrix P, a vector q and a positive constant ε such that

$$\begin{aligned} PA_{cl} + A_{cl}^T P &= -q^T q - \varepsilon P; \\ P[0 \ \cdots \ 0 \ 1]^T &= [\beta C \quad 1]^T. \end{aligned} \qquad (6)$$

Hence, P can be written as

$$P = \begin{bmatrix} P_1 & \beta C^T \\ \beta C & 1 \end{bmatrix}$$

where $P_1 \in \Re^{n \times n}$ is positive-definite. Along the piecewise left-continuous solutions of (4) we define

$$\begin{aligned} V(t) &= [z_p^T(t), z_f(t)] P [z_p^T(t), z_f(t)]^T \\ &= z_p^T(t) P_1 z_p(t) + 2\beta C z_p(t) z_f(t) + z_f^2(t) \end{aligned}$$

over $t \in (t_i, t_{i+1}]$. At the reset instants $t = t_i$ we then have

$$\begin{aligned} V(t_i^+) &= z_p^T(t_i) P_1 z_p(t_i) + 2\beta C z_p(t_i) z_f(t_i^+) + z_f^2(t_i^+) \\ &= V(t_i) + 2\beta C z_p(t_i) z_f(t_i^+) + z_f^2(t_i^+) \\ &\quad -2\beta C z_p(t_i) z_f(t_i) - z_f^2(t_i). \end{aligned}$$

Since $-2\beta C z_p(t_i) z_f(t_i) - z_f^2(t_i) \leq (\beta C z_p(t_i))^2$,

$$\begin{aligned} V(t_i^+) &\leq V(t_i) + 2\beta C z_p(t_i) z_f(t_i^+) + z_f^2(t_i^+) \\ &\quad + (\beta C z_p(t_i))^2 \\ &= V(t_i) + [z_f(t_i^+) + \beta C z_p(t_i)]^2. \end{aligned} \qquad (7)$$

Because w is bounded, it follows from Lemma 2 that there exists a constant $M > 0$ such that $[z_f(t_i^+) + \beta C z_p(t_i)]^2 \leq M$ for $i = 1, 2, \ldots, \infty$. Thus, from (7):

$$V(t_i^+) \leq V(t_i) + M, \quad i = 1, 2, \ldots, \infty.$$

Differentiating $V(t)$ along solutions to (4), we use (6) to obtain

$$\begin{aligned} \dot{V}(t) &= [z_p^T(t), z_f(t)](PA_{cl} + A_{cl}^T P)[z_p^T(t), z_f(t)]^T \\ &= [z_p^T(t), z_f(t)](-q^T q - \varepsilon P)[z_p^T(t), z_f(t)]^T \\ &\leq -\varepsilon [z_p^T(t), z_f(t)] P [z_p^T(t), z_f(t)]^T \\ &= -\varepsilon V(t) \end{aligned}$$

for all $t \in (t_i, t_{i+1}]$. The non-negativity of $V(t)$ implies

$$V(t) \leq e^{-\varepsilon(t - t_i)} V(t_i^+) \qquad (8)$$

whenever $t \in (t_i, t_{i+1}]$. Since $t_{i+1} - t_i > \sigma$,

$$\begin{aligned} V(t_{i+1}) &\leq e^{-\varepsilon(t_{i+1} - t_i)} V(t_i^+) \\ &\leq e^{-\varepsilon\sigma} V(t_i^+) \\ &\leq e^{-\varepsilon\sigma} [V(t_i) + M]. \end{aligned}$$

Combining this with (8) gives

$$\begin{aligned} V(t) &\leq e^{-\varepsilon(t - t_i)} [e^{-\varepsilon(i-1)\sigma} V(0) + M + e^{-\varepsilon\sigma} M \\ &\quad + \ldots + e^{-\varepsilon(i-1)\sigma} M] \end{aligned} \qquad (9)$$

for all $t \in (t_i, t_{i+1}]$. Since $V(0) = 0$, $V(t) \leq M/(1 - e^{-\varepsilon\sigma})$. Therefore, V is bounded. Because P is positive-definite, it follows that z_p is bounded. Finally, from Lemma 1, y is bounded. This completes the proof. □

Remarks: (i) While the β positive-real condition is only sufficient for BIBO stability, it appears that it may be widely applicable to non-trivial situations. For example, in Section 5 we show that this condition is satisfied for a reset control system having 12^{th}-order $L(s)$. Similarly, in [18], the experimental set-up in [6] is shown to satisfy the β positive-real condition (5).

(ii) There exists an important class of reset control systems that satisfy the β positive-real condition and,

[7] A transfer function $X(s)$ is said to be *strictly positive real* if: (i) $X(s)$ is asymptotically stable, and (ii) $\mathrm{Re}[X(j\omega)] > 0, \ \forall \omega \geq 0$.

hence, are BIBO stable. To describe them, consider the reset control systems in Figure 1 with $L(s) = \frac{(s+b)\omega_n^2}{s(s+2\zeta\omega_n)}$ where b is the pole of FORE and $\zeta, \omega_n > 0$. This class was introduced in [3] and its base-linear system has standard second-order transfer function $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$. This class of reset control systems satisfies the β positive-real condition (5) for all combination of positive parameters b, ζ and ω_n; see [9]. Therefore, from Theorem 1, these reset control systems are BIBO stable.

(iii) It is possible that a reset control system is unstable even though its base-linear system is stable and describing-function analysis does not predict a limit-cycle. Such an example is given in [8].

3.1 Robustness to Implementation Errors

In (1) we implicitly assumed that the reset process is ideal; that is, the state of FORE resets exactly to zero at the precise instant when its input $e(t)$ is zero. Of course, this seldom happens as exemplified by the digital implementation of reset elements where such errors occur due to finite sampling rates and signal quantization. To account for such inaccuracies, we modify the model of reset control accordingly to:

$$\begin{aligned}
\dot{x}_p(t) &= Ax_p(t) + Bx_f(t) \\
\dot{x}_f(t) &= -Cx_p(t) - bx_f(t) + w(t); \quad t \notin I \\
x_f(t^+) &= \epsilon_1(t); \qquad\qquad\qquad\qquad t \in I \\
I &= \{t : Cx_p(t) = w(t) + \epsilon_2(t),\ t_{i+1} - t_i > \sigma, \\
&\qquad \sigma > 0, i = 1, 2, ...\}, \qquad\qquad (10)
\end{aligned}$$

where ϵ_1 and ϵ_2 are bounded signals modeling implementation errors. The boundedness of ϵ_2 is necessary for y to be bounded. The following corollary states that the BIBO stability condition in Theorem 1 remains valid even in the face of these implementation errors.

Corollary 1: *The reset control system* (10) *is BIBO stable if it satisfies the β positive-real condition* (5).

Proof: The proof follows along the same lines as that in Theorem 1. After using the state transformation (3), system (10) becomes:

$$\begin{aligned}
\dot{z}_p(t) &= Az_p(t) + Bz_f(t) \\
\dot{z}_f(t) &= -Cz_p(t) - bz_f(t); \quad t \notin I \\
z_f(t_i^+) &= -x_{fl}(t_i) + \epsilon_1(t); \quad t \in I.
\end{aligned}$$

Since ϵ_1 is bounded, it is straightforward to show that Lemma 1 and Lemma 2 are still in effect. Taking the same V and following through the proof of Theorem 1 yields bounded V, z_p, and, finally, bounded y. This completes the proof. □

4 Asymptotic Analysis

In this section we show that satisfaction of the β positive-real condition (5) yields more than BIBO stability. With it, we can further show that the reset control system (1) is *asymptotically stable* and that it inherits the *asymptotic tracking* properties of its base-linear system. In the sequel we denote the tracking error in the reset control system and its base-linear system by $e(t) = w(t) - Cx_p(t)$ and $e_l(t) = w(t) - Cx_{pl}(t)$, respectively. We first need the following technical lemmas.

Lemma 4: *If* $\lim_{t\to\infty} e_l(t) = 0$, *then*

$$\lim_{t\to\infty} Cz_p(t_i) = 0 \ \text{ and } \ \lim_{i\to\infty} z_f(t_i^+) = 0.$$

Proof: From the definition of t_i,

$$Cz_p(t_i) = w(t_i) - Cx_{pl}(t_i) \to 0,$$

as $i \to \infty$. From (2) we have

$$\dot{x}_{fl}(t) = -bx_{fl}(t) - Cx_{pl}(t) + w(t).$$

Since $\lim_{t\to\infty} e_l(t) = 0$ and $b > 0$, then $\lim_{t\to\infty} x_{fl}(t) = 0$. From (4), $z_f(t_i^+) = -x_{fl}(t_i)$ so that $\lim_{i\to\infty} z_f(t_i^+) = 0$. □

Lemma 5: *If the β positive-real condition* (5) *is satisfied and* $\lim_{t\to\infty} e_l(t) = 0$, *then*

$$\lim_{t\to\infty} \left(\begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} - \begin{bmatrix} x_{pl}(t) \\ x_{fl}(t) \end{bmatrix} \right) = 0.$$

Proof: Take $V(t)$ as in the proof of Theorem 1. Then, from (7),

$$V(t_i^+) \le V(t_i) + [z_f(t_i^+) + \beta C z_p(t_i)]^2.$$

With $M_i = [z_f(t_i^+) + \beta C z_p(t_i)]^2$ and $\lim_{t\to\infty} e_l(t) = 0$, it follows from Lemma 4 that $\lim_{i\to\infty} M_i = 0$. Thus, (9) becomes

$$\begin{aligned}
V(t) \ &\le \ e^{-\varepsilon(t-t_i)}[e^{-\varepsilon(i-1)\sigma}V(0) + M_i + e^{-\varepsilon\sigma}M_{i-1} \\
&\quad + \ldots + e^{-\varepsilon(i-1)\sigma}M_1] \qquad (11)
\end{aligned}$$

for all $t \in (t_i, t_{i+1}]$. Since $V(0) = 0$, then from (4) $\lim_{t\to\infty} V(t) = 0$ so that

$$\lim_{t\to\infty} \left(\begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} - \begin{bmatrix} x_{pl}(t) \\ x_{fl}(t) \end{bmatrix} \right) = 0.$$

This completes the proof. □

We now state our asymptotic stability result.

Theorem 2: *The reset control system* (1) *is asymptotically stable if it satisfies the β positive-real condition* (5).

Proof: Set $w(t) \equiv 0$. From (11), it is straightforward to compute

$$V(t) \leq \sup_i \frac{M_i}{1 - e^{-\varepsilon\sigma}}$$

where $M_i = [z_f(t_i^+) + \beta C z_p(t_i)]^2$. From (4), $z_f(t_i^+) = -x_{fl}(t_i)$ and from (3), $Cz_p(t_i) = -Cx_{pl}(t_i)$ so that $M_i = [-x_{fl}(t_i) - \beta C x_{pl}(t_i)]^2$. Therefore,

$$V(t) \leq \sup_{t_i} [-x_{fl}(t_i) - \beta C x_{pl}(t_i)]^2 / (1 - e^{-\varepsilon\sigma}). \quad (12)$$

The right-hand side of (12) can be bounded as in

$$[x_{fl}(t) + \beta C x_{pl}(t)]^2 \leq k \left\| \begin{array}{c} x_{pl}(t) \\ x_{fl}(t) \end{array} \right\|^2$$

for some $k > 0$ and for all $t > 0$. Since $V(t)$ is a positive-definite function, then the left-hand side of (12) can be bounded below by the norm of $[z_p^T(t), z_f(t)]^T$. Hence, there exists a constant k_1 such that

$$\left\| \begin{array}{c} z_p(t) \\ z_f(t) \end{array} \right\| \leq k_1 \sup_{t \in [0,\infty)} \left\| \begin{array}{c} x_{pl}(t) \\ x_{fl}(t) \end{array} \right\|$$

for all $t > 0$. Therefore, from (3),

$$\left\| \begin{array}{c} x_p(t) \\ x_f(t) \end{array} \right\| \leq (k_1 + 1) \sup_{t \in [0,\infty)} \left\| \begin{array}{c} x_{pl}(t) \\ x_{fl}(t) \end{array} \right\|. \quad (13)$$

Since the base-linear system (2) is asymptotically stable and $x_{pl}(0) = x_p(0)$, $x_{fl}(0) = x_f(0)$, then (13) implies that (1) is Lyapunov stable. To complete the proof we need to show that the state asymptotically converges. Since A_{cl} is stable then, from (2),

$$\lim_{t \to \infty} \left[\begin{array}{c} x_{pl}(t) \\ x_{fl}(t) \end{array} \right] = 0.$$

Therefore, from Lemma 5,

$$\lim_{t \to \infty} \left[\begin{array}{c} x_p(t) \\ x_f(t) \end{array} \right] = 0$$

showing that the states asymptotically converge. This proves the theorem. □

We now show that the base-linear system can pass on its asymptotic tracking properties to its reset control system.

Theorem 3: *Suppose the β positive-real condition* (5) *is satisfied. If* $\lim_{t \to \infty} e_l(t) = 0$, *then* $\lim_{t \to \infty} e(t) = 0$.

Proof: From Lemma 5, $\lim_{t \to \infty} [Cx_p(t) - Cx_{pl}(t)] = 0$. Consequently, $\lim_{t \to \infty} e(t) = 0$. This completes the proof. □

Theorem 3 indicates that the classical "type k" behavior of a base-linear system is inherited by its reset control system. Specifically, if $r(t)$ and $d(t)$ are polynomials signals of degree no greater than k, if $lim_{t \to \infty} n(t) = 0$ and if $L(s)$ contains at least k integrators, then the reset system (1) has zero steady-state tracking error provided it satisfies the β positive-real condition (5).

5 Example

In this section we apply the above results to analyze a previously published study on an experimental reset control for a tape-speed servo system [6]. While that reset design achieved improved performance compared with LTI designs, it was not supported with an analysis of stability or steady-state performance. The structure of this system is the one shown in Figure 1 where $L(s)$ is a strictly-proper 14th-order transfer function and the FORE's pole is $b = 30$. To establish stability and steady-state performance we first check the β positive-real condition (5). Since The base-linear system A_{cl} is stable, then $h(s)$ in (5) is asymptotically stable. A simple search and computation shows that $\text{Re}[h(j\omega)] > 0$ for all $\omega \geq 0$ when $\beta = 0.001$; see Figure 2. Invoking Theorems 1-3, we conclude that this reset control system is BIBO and asymptotically stable.

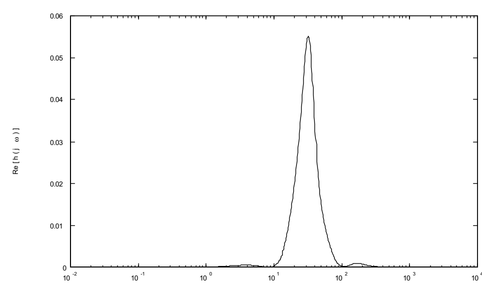

Figure 2: The plot of $Re[h(j\omega)]$.

To analyze steady-state performance, consider the response when $r(t) = 0$ and $n(t) = sin(t)e^{-t}$ shown in Figure 3. As expected from Lemma 4 and 5, we observed a zero steady-state error. Another conformation is shown in Figure 4 where $r(t) = 1$ and $n(t) = -0.1e^{-2t}$.

6 Conclusions

This paper developed a sufficient condition (the β positive-real condition) for BIBO stability for a class of reset control systems. This condition also led to a series of results including asymptotic stability and steady-state performance. The β positive-real condition was shown to be satisfied in an experimental demonstration of reset control, confirming the ob-

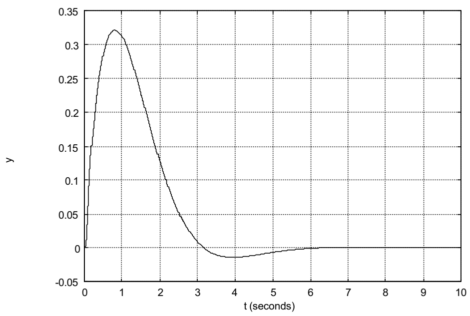

Figure 3: Output $y(t)$ to $r(t) = 0$ and $n(t) = sin(t)e^{-t}$.

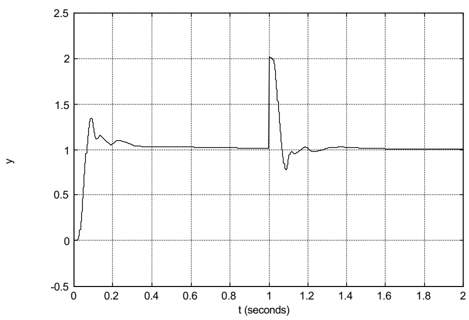

Figure 4: Output $y(t)$ to $r(t) = 1$ and $n(t) = -0.1e^{-2t}$.

served performance as well as demonstrating its applicability.

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